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Electroweak radiative corrections at high energies

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Abstract

We discuss the progress in the calculation of the high-order electroweak radiative corrections to high-energy processes.

Keywords: standard model, electroweak interactions, radiative corrections

1. Introduction

Recently a new wave of interest in the Sudakov asymptotic regime [1, 2] has risen in connection with higher-order corrections to electroweak processes at high energies [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39]. Experimental and theoretical studies of electroweak interactions have traditionally explored the range from very low energies, *e.g.* through parity violation in atoms, up to energies comparable to the masses of the W- and Z-bosons at LEP or the Tevatron. The advent of multi-TeV colliders like the LHC during the present decade or a future linear electron-positron collider will give access to a completely new energy domain. Once the characteristic energies $\sqrt{s}$ are far larger than the masses of the W- and Z-bosons, $M_{W,Z}$, exclusive reactions like electron-positron (or quark-antiquark) annihilation into a pair of fermions or gauge bosons will receive virtual corrections enhanced by powers of the large *electroweak* logarithm $\ln\left(s/M^2_{W,Z}\right)$. The logarithmically enhanced one- and two-loop correction may reach up to 30% and 10% in the TeV region, respectively, and must be included in theoretical predictions for the high precision physics program at the future electron-positron collider. They can be even larger at the energies accessible for the LHC and should be taken into account in the analysis of physics at the hadronic collider. The full evaluation of electroweak one-loop corrections to fermion or gauge boson pair production is by now a straightforward task. Two-loop corrections, however, can be obtained only in the high energy limit. In these Proceedings we review the progress in the study of the dominant two-loop electroweak corrections to various processes at high energy. We focus on the method developed in Refs. [8, 11, 25, 28].

In Sects. 2.1-2.3 we outline the main features of the approach and give a detailed account of its application to a fermion vector form factor in the spontaneously broken gauge theory. The generalization of the method to more complicated processes of the fermion and gauge boson production is discussed in Sects. 2.4, 2.5. In Sect. 3 we describe a method of separating the singular QED contribution and present numerical results for the two-loop electroweak corrections to various processes. Sect. 4 is our conclusion.

2. High energy limit of radiative corrections in spontaneously broken gauge theories

2.1. Vector form factor

The vector form factor $\mathcal{F}$ determines the fermion scattering amplitude in an external Abelian gauge field.

It plays a special role since it is the simplest quantity which includes the complete information about the universal *collinear* logarithms. This information is directly applicable to a process with an arbitrary number of fermions. The form factor is a function of the Euclidean momentum transfer $Q^2 = -(p_1-p_2)^2$ where $p_{1,2}$ is the incoming/outgoing fermion momentum. In the next two sections we consider two characteristic examples: (i) the $SU(2)$ gauge model with gauge bosons of a nonzero mass M which emulates the massive gauge boson sector of the standard model and (ii) the $U(1)\times U(1)$ gauge model with two gauge bosons of essentially different masses which emulates the effect of the $Z-\gamma$ mass gap. We focus on the asymptotic behavior of the form factor in the Sudakov limit $M/Q \ll 1$ with on-shell massless fermions, $p_1^2 = p_2^2 = 0$.

In the Sudakov limit the coefficients of the perturbative series in the coupling constant can be expanded in M^2/Q^2. To compute the leading term of the series in M^2/Q^2 we use the expansion by regions approach [40, 41, 42]. It is based on separating the contributions of dynamical modes or *regions* characteristic for different asymptotic regimes and consists of the following steps:

(i) consider various regions of a loop momentum k and expand, in every region, the integrand in a Taylor series with respect to the parameters considered small in this region;

(ii) integrate the expanded integrand over the whole integration domain of the loop momenta;

(iii) put to zero any scaleless integral.

In step (ii) dimensional regularization with $d = 4 - 2\epsilon$ space-time dimensions is used to handle the divergences. In the Sudakov limit under consideration the following regions are relevant [43, 44, 45]:

$$\begin{aligned}
&\textit{hard}\ (\mathrm{h}): && k \sim Q\,,\\
&\textit{1-collinear}\ (\mathrm{1c}): && k_+ \sim Q,\ k_- \sim M^2/Q\,,\ \underline{k} \sim M\,,\\
&\textit{2-collinear}\ (\mathrm{2c}): && k_- \sim Q,\ k_+ \sim M^2/Q\,,\ \underline{k} \sim M\,,\\
&\textit{soft}\ (\mathrm{s}): && k \sim M\,,
\end{aligned} \tag{1}$$

where $k_\pm = k_0 \pm k_3$, $\underline{k} = (k_1, k_2)$ and we choose $p_{1,2} = (Q/2, 0, 0, \mp Q/2)$ so that $2p_1p_2 = Q^2 = -s$. By $k \sim Q$, *etc.* we mean that every component of k is of order Q.

2.2. $SU(2)$ *model with massive gauge boson*

Let us apply the method to compute the corrections to the form factor in the $SU(2)$ model. In one loop the expansion by regions leads to the following decomposition

$$\mathcal{F}^{(1)} = \mathcal{F}_h^{(1)} + \mathcal{F}_c^{(1)} + \mathcal{F}_s^{(1)}\,, \tag{2}$$

where the subscript c denotes the contribution of both collinear regions. For a perturbative function $f(\alpha)$ we define

$$f(\alpha) = \sum_n \left(\frac{\alpha}{4\pi}\right)^n f^{(n)}\,, \tag{3}$$

and the form factor in the Born approximation is normalized to 1. The hard region contribution, which we will later need, reads

$$\begin{aligned}
\mathcal{F}_h^{(1)} &= \frac{C_F}{Q^{2\epsilon}}\left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{\pi^2}{6} - 8\right.\\
&\quad \left. + \left(-16 + \frac{\pi^2}{4} + \frac{14}{3}\zeta(3)\right)\epsilon\right] + O(\epsilon^2)\,,
\end{aligned} \tag{4}$$

where $C_F = (N^2-1)/(2N)$ for a $SU(N)$ gauge group, $\zeta(3) = 1.202057\ldots$ is the value of the Riemann's zeta-function and all the power-suppressed terms are neglected. For convenience we do not include the standard factor $(4\pi e^{-\gamma_E}(\mu^2))^\epsilon$ per loop, where $\gamma_E = 0.577216\ldots$ is Euler's constant. The contributions of all the regions [11] add up to the well known finite result

$$\mathcal{F}^{(1)} = -C_F\left(\mathcal{L}^2 - 3\mathcal{L} + \frac{7}{2} + \frac{2\pi^2}{3}\right)\,, \tag{5}$$

where $\mathcal{L} = \ln\left(Q^2/M^2\right)$. A similar decomposition can be performed in two loops

$$\mathcal{F}^{(2)} = \mathcal{F}_{hh}^{(2)} + \mathcal{F}_{hc}^{(2)} + \mathcal{F}_{cc}^{(2)} + \ldots \tag{6}$$

The hard-hard part reads

$$\begin{aligned}
\mathcal{F}_{hh}^{(2)} &= \left(\frac{1}{2}\mathcal{F}_h^{(1)} - \frac{\beta_0}{\epsilon}\right)\mathcal{F}_h^{(1)} + \frac{C_F}{Q^{4\epsilon}}\left\{\left[-\frac{11}{6\epsilon^3}\right.\right.\\
&\quad + \left(-\frac{83}{9} + \frac{\pi^2}{6}\right)\frac{1}{\epsilon^2} + \left(-\frac{4129}{108} - \frac{11}{36}\pi^2\right.\\
&\quad \left.\left. + 13\zeta(3)\right)\frac{1}{\epsilon}\right]C_A + \left[\frac{2}{3\epsilon^3} + \frac{28}{9\epsilon^2}\right.\\
&\quad \left. + \left(\frac{353}{27} + \frac{\pi^2}{9}\right)\frac{1}{\epsilon}\right]T_F n_f + \left[\frac{1}{6\epsilon^3} + \frac{17}{18\epsilon^2}\right.\\
&\quad \left. + \left(\frac{455}{108} + \frac{\pi^2}{36}\right)\frac{1}{\epsilon}\right]T_F n_s + \left[-\frac{3}{4} + \pi^2\right.\\
&\quad \left.\left. - 12\zeta(3)\right]\frac{C_F}{\epsilon}\right\} + O(\epsilon^0)\,,
\end{aligned} \tag{7}$$

for α defined in the $\overline{\mathrm{MS}}$ scheme. Here $C_A = N$, $T_F = 1/2$, $\beta_0 = 11C_A/3 - 4T_F n_f/3 - T_F n_s/3$ is the one-loop beta-function, and n_f (n_s) is the number of Dirac fermions (scalars) in the fundamental representation. With α renormalized at the scale M in the one-loop result, the total two-loop contribution takes the form

$$\mathcal{F}^{(2)} = \frac{1}{2}\mathcal{F}^{(1)2} + C_F\left\{\left[\frac{11}{9}\mathcal{L} + \left(-\frac{233}{18} + \frac{\pi^2}{3}\right)\right]C_A + \left[-\frac{4}{9}\mathcal{L} + \frac{38}{9}\right]T_Fn_f + \left[-\frac{1}{9}\mathcal{L} + \frac{25}{18}\right]T_Fn_s\right\}\mathcal{L}^2 + \left[\Delta^{(2)}_{NA} + \Delta^{(2)}_{f} + \Delta^{(2)}_{s} + \Delta^{(2)}_{A}\right]\mathcal{L} + O(\mathcal{L}^0)\,. \quad (8)$$

The coefficients of the second and higher powers of the logarithm are insensitive to the infrared structure of the model as explained below. In contrast, the coefficient of the linear logarithm does in general depend on the whole mass spectrum of the model. The purely Abelian term reads

$$\Delta^{(2)}_A = \left(\frac{3}{2} - 2\pi^2 + 24\zeta(3)\right)C_F^2\,. \quad (9)$$

Massless Dirac fermions give

$$\Delta^{(2)}_f = -\frac{34}{3}C_FT_Fn_f\,. \quad (10)$$

The non-Abelian contribution depends on the details of the gauge boson mass generation. For the spontaneously broken gauge group $SU(2)$ with a single Higgs boson of mass $M_H = M$ in the fundamental representation explicit calculation leads to

$$\Delta^{(2)}_{NA} + \Delta^{(2)}_s = \frac{749}{16} + \frac{43}{24}\pi^2 - 44\zeta(3) + \frac{15}{4}\sqrt{3}\pi + \frac{13}{2}\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\,. \quad (11)$$

Here $\mathrm{Cl}_2\left(\frac{\pi}{3}\right) = 1.014942\ldots$ is the value of the Clausen function. For comparison, in the (hypothetical) case of a light Higgs boson $M_H \ll M$ this contribution becomes

$$\Delta^{(2)}_{NA} + \Delta^{(2)}_s = \frac{747}{16} + \frac{97}{48}\pi^2 - 44\zeta(3) + \frac{33}{8}\sqrt{3}\pi + \frac{21}{4}\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\,. \quad (12)$$

In the electroweak theory inspired model with the $SU(2)_L$ gauge group, six left-handed fermion doublets ($n_f = 3$), and $M_H = M$, the result for the two-loop logarithmic corrections reads [27]

$$\mathcal{F}^{(2)} = \frac{9}{32}\mathcal{L}^4 - \frac{19}{48}\mathcal{L}^3 - \left(\frac{463}{48} - \frac{7}{8}\pi^2\right)\mathcal{L}^2 + \left(29 - \frac{11}{24}\pi^2 - \frac{61}{2}\zeta(3) + \frac{15}{4}\sqrt{3}\pi + \frac{13}{2}\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right)\mathcal{L} + O(\mathcal{L}^0)\,. \quad (13)$$

The asymptotic dependence of the form factor on Q in the Sudakov limit is governed by the hard evolution equation [46, 47, 48]

$$\frac{\partial}{\partial\ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}\,. \quad (14)$$

Its solution is

$$\mathcal{F} = F_0(\alpha(M^2))\exp\left\{\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}\,. \quad (15)$$

By calculating the functions entering the evolution equation order by order in α one gets the logarithmic approximations for the form factor. For example, the LL approximation includes all the terms of the form $\alpha^n\mathcal{L}^{2n}$ and is determined by the one-loop value of $\gamma(\alpha)$; the NLL approximation includes all the terms of the form $\alpha^n\mathcal{L}^{2n-m}$ with $m = 0,\ 1$ and requires the one-loop values of $\gamma(\alpha)$, $\zeta(\alpha)$ and $\xi(\alpha)$ as well as the one-loop running of α in $\gamma(\alpha)$; and so on. The functions entering the evolution equation can in principle be determined by comparing Eq. (15) expanded in the coupling constant to the fixed order result for the form factor. Within the expansion by regions approach the logarithmic contributions show up as singularities of the different regions. One can identify the regions relevant for determining a given parameter of the evolution equation and compute them separately up to the required accuracy which facilitates the analysis by far. For example, the anomalous dimensions $\gamma(\alpha)$ and $\zeta(\alpha)$ are known to be mass-independent and determined by the singularities of the contribution with all the loop momenta being hard [46, 47, 48]. The dimensionally regularized hard contribution exponentiates as well [48, 49] with the functions $\gamma(\alpha)$ and $\zeta(\alpha)$ parameterizing the double and single pole contribution to the exponent. The one- and two-loop hard contribution can be written as

$$\mathcal{F}^{(1)}_h = \frac{1}{Q^{2\epsilon}}\left(\frac{\gamma^{(1)}}{\epsilon^2} - \frac{\zeta^{(1)}}{\epsilon} + F_{0h}^{(1)}\right) + O(\epsilon)\,,$$

$$\mathcal{F}^{(2)}_{hh} = \left(\frac{1}{2}\mathcal{F}^{(1)}_h - \frac{\beta_0}{\epsilon}\right)\mathcal{F}^{(1)}_h + \frac{1}{Q^{4\epsilon}}\left\{\left[\frac{1}{\epsilon^3}\frac{\gamma^{(1)}\beta_0}{4} + \frac{1}{\epsilon^2}\left(\frac{\gamma^{(2)}}{4} - \frac{\zeta^{(1)}\beta_0}{2}\right) + \frac{1}{2\epsilon}\left(-\zeta^{(2)} + F_{0h}^{(1)}\beta_0\right)\right]\right\} + O(\epsilon^0)\,. \quad (16)$$

From Eqs. (4, 7, 16) we find

$$\begin{aligned}
\gamma^{(1)} &= -2C_F\,, \\
\zeta^{(1)} &= 3C_F\,, \\
\gamma^{(2)} &= C_F\left[\left(-\frac{134}{9}+\frac{2}{3}\pi^2\right)C_A+\frac{40}{9}T_Fn_f \right.\\
&\left.\quad +\frac{16}{9}T_Fn_s\right], \\
\zeta^{(2)} &= C_F\left[\left(\frac{2545}{54}+\frac{11}{9}\pi^2-26\zeta(3)\right)C_A \right.\\
&\quad -\left(\frac{418}{27}+\frac{4}{9}\pi^2\right)T_Fn_f-\left(\frac{311}{54}+\frac{\pi^2}{9}\right)T_Fn_s \\
&\left.\quad +\left(\frac{3}{2}-2\pi^2+24\zeta(3)\right)C_F\right]. \qquad (17)
\end{aligned}$$

At the same time the functions $\xi(\alpha)$ and $F_0(\alpha)$ fix the initial conditions for the evolution equation at $Q = M$ and do depend on the infrared structure of the model. To determine the function $\xi(\alpha)$ one has to know the singularities of the collinear region contribution while $F_0(\alpha)$ requires the complete information on the contributions of all the regions. The total one- and two-loop form factor can be expressed through the parameters of the evolution equation as follows

$$\begin{aligned}
\mathcal{F}^{(1)} &= \frac{1}{2}\gamma^{(1)}\mathcal{L}^2+\left(\xi^{(1)}+\zeta^{(1)}\right)\mathcal{L}+F_0^{(1)}\,. \\
\mathcal{F}^{(2)} &= \frac{1}{8}(\gamma^{(1)})^2\mathcal{L}^4+\frac{1}{2}\left(\xi^{(1)}+\zeta^{(1)}-\frac{1}{3}\beta_0\right)\gamma^{(1)}\mathcal{L}^3 \\
&+\frac{1}{2}\left(\gamma^{(2)}+\left(\xi^{(1)}+\zeta^{(1)}\right)^2-\beta_0\zeta^{(1)}+F_0^{(1)}\gamma^{(1)}\right)\mathcal{L}^2 \\
&+\left(\zeta^{(2)}+\xi^{(2)}+F_0^{(1)}(\zeta^{(1)}+\xi^{(1)})\right)\mathcal{L}+O(\mathcal{L}^0)\,. \qquad (18)
\end{aligned}$$

With the known values of $\gamma^{(1,2)}$ and $\zeta^{(1,2)}$ it is straightforward to obtain the result for the remaining functions

$$\begin{aligned}
\xi^{(1)} &= 0\,, \\
F_0^{(1)} &= -C_F\left(\frac{7}{2}+\frac{2\pi^2}{3}\right) \\
\xi^{(2)} &= \xi^{(2)}_{NA}+\xi^{(2)}_f+\xi^{(2)}_s+\xi^{(2)}_A\,, \qquad (19)
\end{aligned}$$

where the Abelian contribution vanishes $\xi^{(2)}_A = 0$, the massless Dirac fermions give

$$\xi^{(2)}_f = \left(\frac{112}{27}+\frac{4}{9}\pi^2\right)C_FT_Fn_f\,, \qquad (20)$$

and for the spontaneously broken $SU(2)$ model with $M_H = M$ we get

$$\begin{aligned}
\xi^{(2)}_{NA}+\xi^{(2)}_s &= -\frac{391}{18}-5\zeta(3)+\frac{15}{4}\sqrt{3}\pi \\
&\quad +\frac{13}{2}\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right). \qquad (21)
\end{aligned}$$

Note that the functions $\gamma(\alpha)$ and $\xi(\alpha)$ are protected against the Abelian multiloop corrections by the properties of the light-cone Wilson loop [47, 48, 50].

The analysis of the evolution equation gives a lot of insight into the structure of the logarithmic corrections. For example, Eq. (18) tells us that, up to the NNLL approximation, the information on the infrared structure of the model enters through the one-loop coefficients $\xi^{(1)}$ and $F_0^{(1)}$ which are insensitive to the details of the mass generation. Thus one can compute the coefficient of the two-loop quadratic logarithm with the gauge boson mass introduced by hand [11].

2.3. $U(1) \times U(1)$ *model with mass gap*

Let us now discuss the second example, a $U(1)\times U(1)$ model with λ, α' and M, α for masses and coupling constants, respectively. We consider the limit $\lambda \ll M$ and make use of the *infrared* evolution equation which governs the dependence of the form factor $\mathcal{F}(\lambda, M, Q)$ on λ [7]. The virtual corrections become divergent in the limit $\lambda \to 0$. According to the Kinoshita-Lee-Nauenberg theorem [51, 52], these divergences are cancelled against the ones of the corrections due to the emission of real light gauge bosons of vanishing energy and/or collinear to one of the on-shell fermion lines. The singular behavior of the form factor must be the same in the full $U_{\alpha'}(1) \times U_\alpha(1)$ theory and the effective $U_{\alpha'}(1)$ model with only the light gauge boson. For $\lambda \ll M \ll Q$ the solution of the infrared evolution equation is given by the Abelian part of the exponent (15) with M, α replaced by λ, α'. Thus the form factor can be written in a factorized form

$$\mathcal{F}(\lambda, M, Q) = \tilde{F}(M, Q)\mathcal{F}_{\alpha'}(\lambda, Q) + O(\lambda/M)\,, \qquad (22)$$

where $\mathcal{F}_{\alpha'}(\lambda, Q)$ stands for the $U_{\alpha'}(1)$ form factor and $\tilde{F}(M, Q)$ depends both on α and α', and incorporates all the logarithms of the form $\ln\left(Q^2/M^2\right)$. It can be obtained directly by calculating the ratio

$$\tilde{F}(M, Q) = \left[\frac{\mathcal{F}(\lambda, M, Q)}{\mathcal{F}_{\alpha'}(\lambda, Q)}\right]_{\lambda\to 0}. \qquad (23)$$

Since the function $\tilde{F}(M, Q)$ does not depend on the infrared regularization, the ratio in Eq. (23) can be evaluated with $\lambda = 0$ using dimensional regularization for the infrared divergences. The resulting two-parameter perturbative expansion is

$$\tilde{F}(M, Q) = \sum_{n,m}\frac{\alpha'^n\alpha^m}{(4\pi)^{n+m}}\tilde{F}^{(n,m)} \qquad (24)$$

where

$$\tilde{F}^{(0,0)} = 1, \qquad \tilde{F}^{(n,0)} = 0, \qquad \tilde{F}^{(0,m)} = \mathcal{F}^{(m)}\,, \qquad (25)$$

and the two-loop interference term reads [25]

$$\tilde{F}^{(1,1)} = \left(3 - 4\pi^2 + 48\zeta(3)\right)\mathcal{L} + O(\mathcal{L}^0). \quad (26)$$

In the equal mass case, $\lambda = M$, we have an additional reparameterization symmetry, and the form factor is determined by Eq. (15) with the effective coupling $\bar{\alpha} = \alpha' + \alpha$ so that $\mathcal{F}(M, M, Q) = \mathcal{F}_{\bar{\alpha}}(M, Q)$. We can now write down the matching relation

$$\mathcal{F}(M, M, Q) = C(M, Q)\tilde{F}(M, Q)\mathcal{F}_{\alpha'}(M, Q), \quad (27)$$

where the matching coefficient $C(M, Q)$ represents the effect of the power-suppressed terms neglected in Eq. (22). By combining the explicit results for $\mathcal{F}_{\alpha'}(M, Q)$ and $\tilde{F}(M, Q)$ the matching coefficient was found to be $C(M, Q) = 1 + O(\alpha'\alpha\mathcal{L}^0)$ [25]. In two-loops it does not contain logarithmic terms, and up to the N^{3}LL accuracy, the product $\tilde{F}(M, Q)\mathcal{F}_{\alpha'}(\lambda, Q)$ continuously approaches $\mathcal{F}(M, M, Q)$ as λ goes to M. Therefore, to get *all* the logarithms of the heavy gauge boson mass in two-loop approximation for the theory with mass gap, it is sufficient to divide the form factor $\mathcal{F}_{\bar{\alpha}}(M, Q)$ of the symmetric phase by the form factor $\mathcal{F}_{\alpha'}(\lambda, Q)$ of the effective $U_{\alpha'}(1)$ theory taken at the symmetric point $\lambda = M$. Thus we have reduced the calculation in the theory with mass gap to the one in the symmetric theory with a single mass parameter. The logarithmic terms in the expansion of $\tilde{F}(M, Q)$ exponentiate by construction and one can describe the exponent with a set of functions $\tilde{\gamma}(\alpha, \alpha')$, $\tilde{\zeta}(\alpha, \alpha')$, $\tilde{\xi}(\alpha, \alpha')$ and $\tilde{F}_0(\alpha, \alpha')$ in analogy with Eq. (15). The matching procedure can naturally be formulated in terms of these functions. For the mass-independent functions we have the all order relation

$$\begin{aligned} \tilde{\gamma}(\alpha', \alpha) &= \gamma(\bar{\alpha}) - \gamma(\alpha'), \\ \tilde{\zeta}(\alpha', \alpha) &= \zeta(\bar{\alpha}) - \zeta(\alpha'). \end{aligned} \quad (28)$$

In two loops we obtain by explicit calculation

$$\tilde{\xi}(\alpha', \alpha) = \xi(\bar{\alpha}) = 0, \quad (29)$$

which holds in higher orders for the Abelian model due to the nonrenormalization properties discussed in the previous section. Thus, the only nontrivial two-loop matching is for the coefficient $\tilde{F}_0^{(1,1)}$ due to the non-logarithmic contribution to $C(M, Q)$ which is beyond the accuracy of our analysis.

Note that the absence of the two-loop linear-logarithmic term in $C(M, Q)$ is an exceptional feature of the Abelian corrections. The general analysis of the evolution equation [11] shows that the terms neglected in Eq. (22) contribute starting from the N^{3}LL approximation. Indeed, the solution of the hard evolution equation for $F(\lambda, M, Q)$ which determines its dependence on Q is of the form (15) with the infrared sensitive quantities F_0 and ξ being functions of the ratio λ/M. A nontrivial dependence on the mass ratio in general emerges first through the two-loop coefficient $\xi^{(2)}$ due to the interference diagrams with both massive and massless gauge bosons. The matching is necessary to take care of the difference $\xi^{(2)}|_{\lambda/M=1} \neq \xi^{(2)}|_{\lambda/M=0}$. Thus, for a non-Abelian theory with the mass gap of the standard model type, *i.e.* with interaction between the heavy and light gauge bosons, the matching becomes nontrivial already in N^{3}LL approximation.

2.4. Four-fermion process

We consider the four-fermion scattering at fixed angles in the limit where all the kinematical invariants are of the same order and are far larger than the gauge boson mass, $|s| \sim |t| \sim |u| \gg M^2$. The analysis of the four-fermion amplitude is complicated by the additional kinematical variable and the presence of different isospin and Lorentz structures. The collinear divergences in the hard part of the virtual corrections and the corresponding *collinear* logarithms are known to factorize. They are responsible, in particular, for the double logarithmic contribution and depend only on the properties of the external on-shell particles but not on the specific process [46, 47, 48, 53, 54, 55]. This fact is especially clear if a physical (Coulomb or axial) gauge is used for the calculation. In this gauge the collinear divergences are present only in the self energy insertions to the external particles. Thus, for each fermion-antifermion pair of the four-fermion amplitude the collinear logarithms are the same as for the form factor $\mathcal{F}$ discussed in the previous section. Let us denote by $\tilde{\mathcal{A}}$ the amplitude with the collinear logarithms factored out. For convenience we separate from $\tilde{\mathcal{A}}$ all the corrections entering Eq. (15) so that

$$\mathcal{A} = \frac{ig^2}{s}\mathcal{F}^2\tilde{\mathcal{A}}. \quad (30)$$

The resulting amplitude $\tilde{\mathcal{A}}$ contains the logarithms of the *soft* nature corresponding to the soft divergences of the hard region contribution and the renormalization group logarithms. It can be represented as a vector in the color/chiral basis and satisfies the following evolution equation [55, 56, 57]:

$$\frac{\partial}{\partial \ln Q^2}\tilde{\mathcal{A}} = \chi(\alpha(Q^2))\tilde{\mathcal{A}}, \quad (31)$$

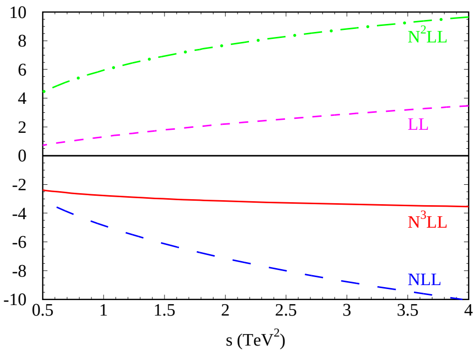

Figure 1: Logarithmic contributions to $\sigma(e^+e^- \to q\bar{q})$ in % to the Born approximation: the two-loop LL ($\ln^4(s/M^2)$, short-dashed line), NLL ($\ln^3(s/M^2)$, long-dashed line), NNLL ($\ln^2(s/M^2)$, dot-dashed line), and N^{3}LL ($\ln^1(s/M^2)$, solid line) terms [28].

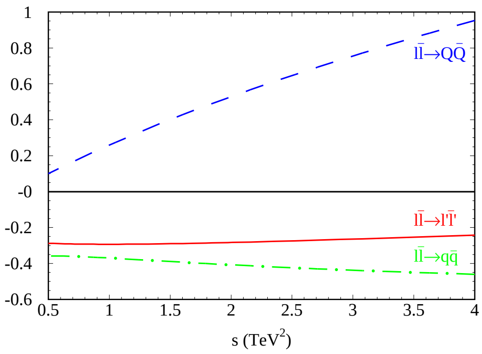

Figure 2: The total two-loop logarithmic corrections to $\sigma(e^+e^- \to Q\bar{Q})$ (dashed line), $\sigma(e^+e^- \to q\bar{q})$ (dot-dashed line) and $R(e^+e^- \to \mu^+\mu^-)$ (solid line) in % to the Born approximation [28].

where $\chi(\alpha)$ is the matrix of the soft anomalous dimensions. The solution of Eq. (31) is given by the path-ordered exponent

$$\tilde{\mathcal{A}} = \mathrm{Pexp}\left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\chi(\alpha(x))\right]\tilde{\mathcal{A}}_0(\alpha(M^2))\,, \qquad (32)$$

where $\tilde{\mathcal{A}}_0(\alpha)$ determines the initial conditions for the evolution equation at $Q = M$. The matrix of the soft anomalous dimensions is determined by the coefficients of the single pole of the hard region contribution to the exponent (32).

Let us again discuss the standard model inspired example considered in the previous section. With the result for the amplitudes it is straightforward to compute the two-loop corrections to the total cross section of the four-fermion annihilation process. For the annihilation process $f\bar{f} \to f'\bar{f}'$ one has to make the analytical continuation of the above result to the Minkowskian region of negative $Q^2 = -s$ according to the $s + i0$ prescription. The above approximation is formally not valid for the small angle region $\theta < M/\sqrt{s}$, which, however, gives only a power-suppressed contribution to the total cross section. For the $SU(2)_L$ model we obtain [27]

$$\sigma^{(2)} = \left[\frac{9}{2}\mathcal{L}^4(s) - \frac{449}{6}\mathcal{L}^3(s) + \left(\frac{4855}{18} + \frac{37}{3}\pi^2\right)\mathcal{L}^2(s) + \left(\frac{48049}{216} - \frac{1679}{18}\pi^2 - 122\zeta(3) + 15\sqrt{3}\pi + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right)\mathcal{L}(s)\right]\sigma_B\,, \qquad (33)$$

and

$$\sigma^{(2)} = \left[\frac{9}{2}\mathcal{L}^4(s) - \frac{125}{6}\mathcal{L}^3(s) - \left(\frac{799}{9} - \frac{37}{3}\pi^2\right)\mathcal{L}^2(s) + \left(\frac{38005}{216} - \frac{383}{18}\pi^2 - 122\zeta(3) + 15\sqrt{3}\pi + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right)\mathcal{L}(s)\right]\sigma_B\,, \qquad (34)$$

for the initial and final state fermions of the same or opposite isospin, respectively. Here σ_B is the Born cross section with the $\overline{\mathrm{MS}}$ coupling constant renormalized at the scale $\sqrt{s}$ and $\mathcal{L}(s) = \ln(s/M^2)$.

2.5. Gauge boson production

In this section we consider the production of a pair of massive gauge bosons in annihilation of the fermion-antifermion pair. To be specific we focus on the process with a given initial and final isospin states corresponding to $e^+e^- \to W^+W^-$. Due to helicity conservation a pair of either transverse or longitudinal gauge bosons can be produced in the high energy limit. The transverse gauge bosons behave like vector particles in the adjoint representation while the longitudinal gauge bosons, as a consequence of the equivalence theorem, behave like scalar particles in the fundamental representation. The structure of the Sudakov logarithms in these cases is significantly different and we consider them separately.

The transverse gauge bosons are the true vector particles. The Born amplitude in this case is given by the t-channel fermion exchange diagrams and the Born cross section is peaked in the forward direction. In the $SU_L(2)$ model introduced in Sect. 2.1 with $M_H = M$ and twelve left-handed fermion doublets we have the two-loop logarithmic corrections up to the NNLL terms [31, 38]

$$\frac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}\sigma_B} = \frac{121}{8}\mathcal{L}^4(s) + \left[\left(44 - \frac{22x_-}{x_+}\right)\ln(x_-) - 22\ln(x_+) - \frac{341}{18}\right]\mathcal{L}^3(s)$$

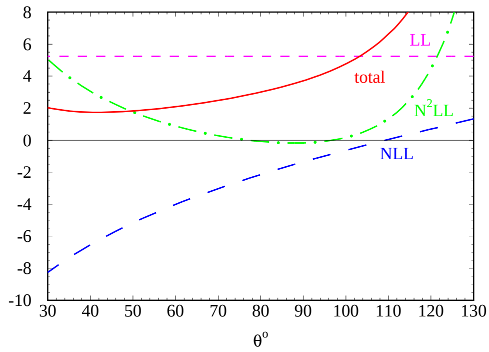

Figure 3: The two-loop logarithmic corrections to $d\sigma(e^+e^- \to W_T^+\bar{W}_T^-)/d\Omega$ in % to the Born approximation: the two-loop LL ($\ln^4(s/M^2)$, short-dashed line), NLL ($\ln^3(s/M^2)$, long-dashed line), and NNLL ($\ln^2(s/M^2)$, dot-dashed line) terms, at $\sqrt{s} = 1$ TeV as functions of the production angle [31, 38].

$$+\left[\left(32+\frac{4x_-^2}{x_+^2}-\frac{55+33x_-}{4(x_-^2+x_+^2)}+\frac{55-40x_-}{2x_+}\right)\ln^2(x_-)\right.$$
$$-\left(28+\frac{22-4x_-}{x_+}\right)\ln(x_-)\ln(x_+)$$
$$+\left(-\frac{70}{3}+\frac{35x_-}{3x_+}-\frac{99-209x_-}{4(x_-^2+x_+^2)}\right)\ln(x_-)$$
$$+\frac{35}{3}\ln(x_+)+\frac{55x_+}{4(x_-^2+x_+^2)}+\frac{209\pi^2}{36}$$
$$\left.+\frac{143\pi}{6\sqrt{3}}-\frac{863}{24}+4\pi^2\frac{x_-^2-x_+^2}{x_+^2}\right]\mathcal{L}^2(s)\,, \qquad (35)$$

where $x_\pm = (1 \pm \cos\theta)/2$, θ is the production angle and the coupling constant in the Born cross section is renormalized at the scale M. Note that in contrast to the four-fermion processes, the cross section of the gauge boson production depends on the Higgs boson mass already in the NNLL approximation.

The equivalence theorem relates the amplitude of the longitudinal gauge boson production $e^+e^- \to W_L^+W_L^-$ to the production of the Goldstone bosons $e^+e^- \to \phi^+\phi^-$. The Born amplitude is now given by the s-channel annihilation diagram and the Born cross section has a maximum at $\theta = 90^o$. If we neglect the quark masses the result for the two-loop corrections reads [31]

$$\frac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}\sigma_B} = \frac{9}{2}\mathcal{L}^2(s)+\left[30\ln(x_-)-6\ln(x_+)-\frac{85}{3}\right]\mathcal{L}^3(s)$$
$$+\left[\left(38+\frac{15}{2x_+}\right)\ln^2(x_-)+\left(2-\frac{3}{2x_-}\right)\ln^2(x_+)\right.$$
$$-8\ln(x_-)\ln(x_+)-\frac{535}{6}\ln(x_-)+\frac{107}{6}\ln(x_+)$$
$$\left.+9\pi^2-\frac{32\pi}{\sqrt{3}}+\frac{229}{4}\right]\mathcal{L}^2(s)\,, \qquad (36)$$

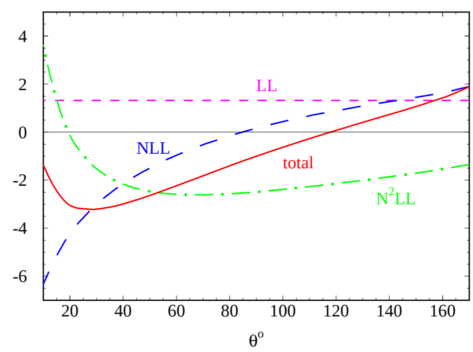

Figure 4: The same as Fig. 3 but for $d\sigma(e^+e^- \to W_L^+\bar{W}_L^-)/d\Omega$ [31].

where the coupling constant in σ_B is renormalized at $\sqrt{s}$.

In the case of longitudinal polarization the large Yukawa coupling of the third generation quarks to the scalar (Higgs and Goldstone) bosons results in specific logarithmic corrections proportional to m_t^2/M_W^2. The high energy evolution of the form factors in a theory with Yukawa interaction is completely analogous to the one of ϕ^3 scalar theory in six dimensions [58]. The structure of factorization and evolution equations is much simpler than in a gauge theory because Yukawa interaction itself does not contribute to the anomalous dimension $\gamma(\alpha)$ and results only in single logarithmic corrections completely determined by the ultraviolet field renormalization of the external on-shell particles. These corrections can be taken into account through the modification of the evolution equations for the form factors. The analysis is straightforward but complicated because the Yukawa interaction mixes evolution of the quark and scalar boson form factors and in general does not commute with the $SU(2)$ and hypercharge couplings. However, due to the factorization of the double Sudakov logarithms, the Yukawa enhanced contribution to NLL approximation is given simply by the product of the one-loop Yukawa corrections and the double logarithmic exponent as observed in Ref. [15]. The structure of the NNLL contribution is much more complicated. The two-loop NNLL Yukawa enhanced contribution to the cross section in the $SU(2)$ model reads [31]

$$\left.\frac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}\sigma_B}\right|_{Yuk}^{NNLL} = \left[-\frac{3}{4}\frac{m_t^4}{M_W^4}+\frac{m_t^2}{M_W^2}\left(30\ln(x_-)\right.\right.$$
$$\left.\left.-6\ln(x_+)-\frac{45}{4}\right)\right]\mathcal{L}^2(s)\,. \qquad (37)$$

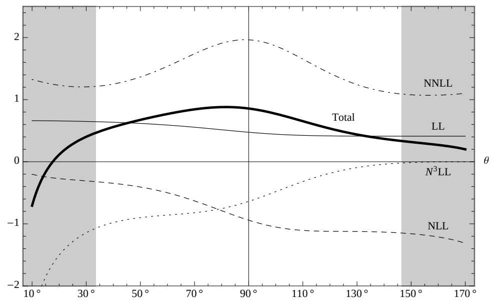

Figure 5: Two-loop logarithmic contributions to the differential Bhabha cross sectionin % to the Born approximation as functions of the scattering angle for $\sqrt{s} = 1$ TeV The shaded area corresponds to the region where the Sudakov approximation is not reliable [39].

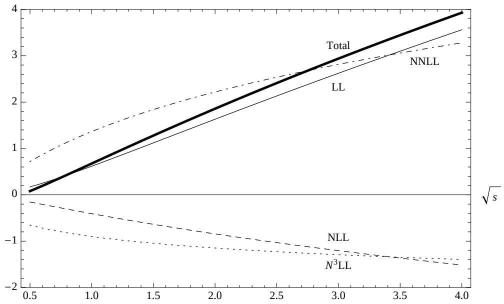

Figure 6: Two-loop logarithmic contributions to the differential Bhabha cross section in % to the Born approximation as functions of the center-of-mass energy for $\theta = 50°$ [39].

3. Two-loop electroweak corrections

The calculation of the two-loop electroweak corrections even in the high energy limit is a challenging theoretical problem at the limit of available computational techniques. It is complicated in particular by the presence of the mass gap and mixing in the gauge sector. Below we describe the approach of Ref. [25] which reduces the analysis of the dominant two-loop logarithmic electroweak corrections to a problem with a single mass parameter which has been solved in the previous section.

3.1. Separating QED infrared logarithms

The main difference between the analysis of the electroweak standard model with the spontaneously broken $SU_L(2)\times U(1)$ gauge group and the treatment of the pure $SU_L(2)$ case considered above is the presence of the massless photon which results in infrared divergences of fully exclusive cross sections. We regularize these divergences by giving the photon a small mass λ. The dependence of the virtual corrections on λ in the limit $\lambda^2 \ll M^2 \ll Q^2$ is governed by the QED infrared evolution equation. For example, for the $f\bar{f} \to f'\bar{f}'$ process it is given by the factor

$$\mathcal{U} = U_0(\alpha_e(Q^2)) \exp\left\{ \frac{\alpha_e(\lambda^2)}{4\pi} \left[-\left(Q_f^2 + Q_{f'}^2\right) \ln^2\left(\frac{Q^2}{\lambda^2}\right) + \left(3\left(Q_f^2 + Q_{f'}^2\right) + 4\ln\left(\frac{x_+}{x_-}\right) Q_{f'} Q_f\right) \ln\left(\frac{Q^2}{\lambda^2}\right) + \mathcal{O}(\alpha_e^2)\right]\right\}, \tag{38}$$

where α_e is the $\overline{\rm MS}$ QED coupling constant and Q_f is the electric charge of the fermion.

Our goal now is to separate the above infrared divergent QED contribution from the total two-loop corrections to get the pure electroweak logarithms $\ln(Q^2/M_{W,Z}^2)$. Within the evolution equation approach [11] it has been found that the electroweak and QED logarithms up to the NNLL approximation can be disentangled by means of the following two-step procedure:

(i) the corrections are evaluated using the fields of the unbroken symmetry phase with all the gauge bosons of the same mass $M \approx M_{Z,W}$, *i.e.* without mass gap;

(ii) the QED contribution (38) with $\lambda = M$ is factorized leaving the pure electroweak logarithms.

This reduces the calculation of the two-loop electroweak logarithms up to the quadratic term to a problem with a single mass parameter. Then the effect of the $Z - W$ boson mass splitting can systematically be taken into account within an expansion around the equal mass approximation [25]. In general the above two-step procedure is not valid in the N^{3}LL approximation which is sensitive to fine details of the gauge boson mass generation. For the exact calculation of the coefficient of the two-loop linear-logarithmic term one has to use the true mass eigenstates of the standard model. The evaluation of the corrections in this case becomes a very complicated multiscale problem. The analysis, however, is drastically simplified in a model with a Higgs boson of zero hypercharge. In this model the mixing is absent and the above two-step procedure can be applied to disentangle all the two-loop logarithms of the $SU_L(2)$ gauge boson mass from the infrared logarithms associated with the massless hypercharge gauge boson (see the discussion below). In the standard model the mixing of the gauge bosons results in a linear-logarithmic contribution which is not accounted for in this approximation. It is, however, suppressed by the small factor $\sin^2\theta_W \approx 0.2$. Therefore, the approximation gives an estimate of the coefficient in front of the linear electroweak logarithm with 20% accuracy. As we will see, a 20% error in the coefficient in front of the two-loop linear electroweak logarithm leads to an uncertainty comparable to the nonlogarithmic contribution and is practically negligible. If we also neglect the difference between M_H and $M_{Z,W}$, the calculation involves a single mass parameter at every step and the results of the previous sections can directly be applied to the isospin $SU(2)_L$ gauge group with the coupling g and the hypercharge $U(1)$ gauge group with the coupling $\tan\theta_W g$.

3.2. Numerical results

In this section we present the numerical results for the dominant two-loop electroweak corrections to a number of benchmark processes at a future electron-positron collider obtained in Refs. [28, 31, 39]. In Fig. 1 the values of different logarithmic contributions to the total cross section of the quark-antiquark pair production cross section $\sigma(e^+e^- \to q\bar{q})$ for $q = d,\ s,\ b$ are plotted separately as functions of s. The total two-loop logarithmically enhanced corrections to $\sigma(e^+e^- \to \mu^+\mu^-)$, $\sigma(e^+e^- \to q\bar{q})$, and $\sigma(e^+e^- \to Q\bar{Q})$, where $Q = u,\ c$, are given in Fig. 2. The two-loop logarithmic corrections up to the NNLL order are plotted in Figs. 3, 4 for the differential cross sections of the transverse and longitudinal W-boson pair production for $\sqrt{s} = 1$ TeV as functions of the production angle. The energy and angular dependence of the two-loop logarithmic corrections to the differential cross section of the electron-positron Bhabha scattering are presented in Figs. 5, 6.

4. Summary

We have reviewed a method [8, 11, 25, 28] of calculation of the dominant high-order electroweak radiative corrections to processes at the energies above the electroweak scale. The method relies on the knowledge of the general infrared structure of gauge theories. It turns out to be very useful not only for the analysis of the logarithmic corrections in spontaneously broken gauge models, but also for the high-order calculations in QED with massive fermions. In particular, the infrared matching procedure based on the same idea has been instrumental for the calculation of the two-loop photonic and heavy flavor corrections to the Bhabha scattering [59, 60, 61, 62]. Though in these Proceedings we focus mainly on the applications relevant for a future high-energy electron-positron collider, many results have been obtained also for the physics at the LHC [29, 32, 38]. Comparison to the one-loop results which retain the full mass dependence [63, 64] confirm that the logarithmic approximation works very well in the region where the electroweak corrections become important.

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