Stable alignment limit in 2HDM

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Based on

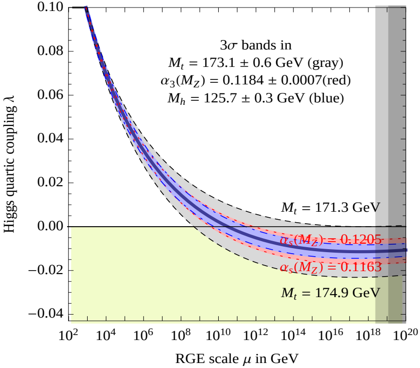
1. **Search for a stable alignment limit in two-Higgs-doublet models**,
   D. Das and **IS**
   Phys.Rev. D91 (2015) 9, 095024.
2. **Distinguishing Neutrino Mass Hierarchies using Dark Matter Annihilation Signals at IceCube**,
   R. Allahverdi, B. Dutta, D. K. Ghosh, B. Knockel and **IS**
   [arXiv:1506.08285 [hep-ph]] (Accepted in JCAP)

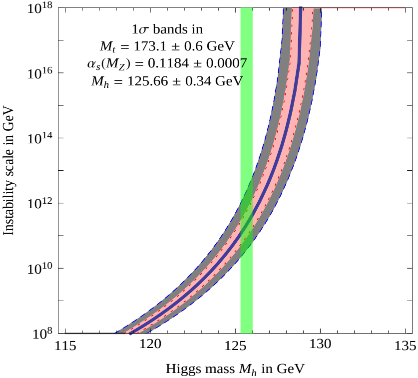
BSM

as a solution to

Vacuum Stability Problem

Vacuum Stability Problem





[Degrassi et al. '2012].

$$m_h > 129\text{ GeV} + 2.0\,(M_t[\text{GeV}] - 173.34\text{ GeV}) - 0.5\text{ GeV}\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \pm 0.3\text{ GeV}\,.$$

[Buttazzo et al. '2013].

Two-Higgs-Doublet Models

Introduction :

- Introduce second Higgs doublet. [Branco et al '11].
- Electroweak ρ- parameter remains unity at tree level.
- FCNC is possible at tree level $\Rightarrow$ need some additional discrete (Z_2) or continuous $(U(1))$ symmetry to prevent.
- $\phi_1 \to \phi_1\,, \phi_2 \to -\phi_2$ leads to four variants:
  Type I : All quarks and leptons couple to ϕ_2.
  Type II : ϕ_2 couples to up type and ϕ_1 couples to down type quarks and charged leptons.
  Type X or lepton-specific : All quarks couple to ϕ_2 while ϕ_1 couples to charged leptons.
  Type Y or flipped : Up type quarks and charged leptons couple to ϕ_2 and down type quarks couple to ϕ_1.
- Good assumption : *alignment limit*.

Two-Higgs-Doublet Models

Potential :

- Notation-I :

$$V = m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 - \left(m_{12}^2\phi_1^\dagger\phi_2 + \text{h.c.}\right) + \frac{\lambda_1}{2}\left(\phi_1^\dagger\phi_1\right)^2 + \frac{\lambda_2}{2}\left(\phi_2^\dagger\phi_2\right)^2$$
$$+\lambda_3\left(\phi_1^\dagger\phi_1\right)\left(\phi_2^\dagger\phi_2\right) + \lambda_4\left(\phi_1^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_1\right) + \left\{\frac{\lambda_5}{2}\left(\phi_1^\dagger\phi_2\right)^2 + \text{h.c.}\right\}$$

Two-Higgs-Doublet Models

Potential :

- Notation-I :

$$
\begin{aligned}
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&+ \lambda_3 \left(\phi_1^\dagger \phi_1 \right) \left(\phi_2^\dagger \phi_2 \right) + \lambda_4 \left(\phi_1^\dagger \phi_2 \right) \left(\phi_2^\dagger \phi_1 \right) + \left\{ \frac{\lambda_5}{2} \left(\phi_1^\dagger \phi_2 \right)^2 + \text{h.c.} \right\}
\end{aligned}
$$

- Notation-II :

$$
\begin{aligned}
V &= \beta_1 \left(\phi_1^\dagger \phi_1 - \frac{v_1^2}{2} \right)^2 + \beta_2 \left(\phi_2^\dagger \phi_2 - \frac{v_2^2}{2} \right)^2 + \beta_3 \left(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 - \frac{v_1^2 + v_2^2}{2} \right) \\
&+ \beta_4 \left\{ (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) - (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \right\} + \beta_5 \left(\text{Re } \phi_1^\dagger \phi_2 - \frac{v_1 v_2}{2} \right)^2 \\
&+ \beta_6 \left(\text{Im } \phi_1^\dagger \phi_2 \right)^2 .
\end{aligned}
$$

Two-Higgs-Doublet Models

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$$+\lambda_3\left(\phi_1^\dagger\phi_1\right)\left(\phi_2^\dagger\phi_2\right) + \lambda_4\left(\phi_1^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_1\right) + \left\{\frac{\lambda_5}{2}\left(\phi_1^\dagger\phi_2\right)^2 + \text{h.c.}\right\}$$

- Notation-II :

$$V = \beta_1\left(\phi_1^\dagger\phi_1 - \frac{v_1^2}{2}\right)^2 + \beta_2\left(\phi_2^\dagger\phi_2 - \frac{v_2^2}{2}\right)^2 + \beta_3\left(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 - \frac{v_1^2+v_2^2}{2}\right)$$
$$+\beta_4\left\{(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)\right\} + \beta_5\left(\text{Re}\ \phi_1^\dagger\phi_2 - \frac{v_1v_2}{2}\right)^2$$
$$+\beta_6\left(\text{Im}\ \phi_1^\dagger\phi_2\right)^2 .$$

- The additional symmetry is softly broken by the term proportional to $\Rightarrow \beta_5$ or m_{12}^2.
- Nonzero $\tan\beta$ implies two parametrizations are equivalent.
- Notation-II is useful for tracking breaking parameter effect and defining scalar masses in terms of couplings.
- The symmetry enhanced from Z_2 to a $U(1)$ for $\beta_5 = \beta_6$.

Two-Higgs-Doublet models

- The equivalence of the two sets of parameters are given by the following relations:

$$m_{11}^2 = -(\beta_1 v_1^2 + \beta_3 v^2)\ ; \qquad \lambda_1 = 2(\beta_1 + \beta_3)\ ;$$
$$m_{22}^2 = -(\beta_2 v_2^2 + \beta_3 v^2)\ ; \qquad \lambda_2 = 2(\beta_2 + \beta_3)\ ;$$
$$m_{12}^2 = \frac{\beta_5}{2} v_1 v_2\ ; \qquad \lambda_3 = (2\beta_3 + \beta_4)\ ;$$
$$\lambda_4 = \frac{\beta_5 + \beta_6}{2} - \beta_4\ ; \qquad \lambda_5 = \frac{\beta_5 - \beta_6}{2}\ .$$

- Five independent parameters : $(m_H, m_A, m_{H^\pm}, \tan\beta, \beta_5)$ where, β_5 is the soft-symmetry breaking parameter.
- The connection between the couplings to the masses is demonstrated by the following relations:

$$\beta_1 = \frac{1}{2v^2 c_\beta^2}\left[m_H^2 c_\alpha^2 + m_h^2 s_\alpha^2 - \frac{s_\alpha c_\alpha}{\tan\beta}\left(m_H^2 - m_h^2\right)\right] - \frac{\beta_5}{4}\left(\tan^2\beta - 1\right)\ ,$$

$$\beta_2 = \frac{1}{2v^2 s_\beta^2}\left[m_h^2 c_\alpha^2 + m_H^2 s_\alpha^2 - s_\alpha c_\alpha \tan\beta\left(m_H^2 - m_h^2\right)\right] - \frac{\beta_5}{4}\left(\cot^2\beta - 1\right)\ ,$$

$$\beta_3 = \frac{1}{2v^2}\frac{s_\alpha c_\alpha}{s_\beta c_\beta}\left(m_H^2 - m_h^2\right) - \frac{\beta_5}{4}\ ,$$

$$\beta_4 = \frac{2}{v^2} m_{H^+}^2\ , \beta_6 = \frac{2}{v^2} m_A^2\ .$$

Theoretical Constraints

- Vacuum stability :

$$\lambda_1 > 0,$$
$$\lambda_2 > 0,$$
$$\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0,$$
$$\lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1 \lambda_2} > 0.$$

- Unitarity :

$$\begin{aligned}
a_1^{\pm} &= \frac{3}{2}(\lambda_1 + \lambda_2) \pm \sqrt{\frac{9}{4}(\lambda_1 - \lambda_2)^2 + (2\lambda_3 + \lambda_4)^2}\,, \\
a_2^{\pm} &= \frac{1}{2}(\lambda_1 + \lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_4^2}\,, \\
a_3^{\pm} &= \frac{1}{2}(\lambda_1 + \lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_5^2}\,, \\
b_1^{\pm} &= \lambda_3 + 2\lambda_4 \pm 3\lambda_5\,, \\
b_2^{\pm} &= \lambda_3 \pm \lambda_5\,, \\
b_3^{\pm} &= \lambda_3 \pm \lambda_4\,.
\end{aligned}$$

The requirement of tree unitarity then restricts the above eigenvalues as follows:

$$|a_i^{\pm}|\,,\ |b_i^{\pm}| \leq 16\pi\,.$$

Experimental Constraints

- Oblique T - parameter constraint $\Rightarrow$ restrict the splitting between nonstandard masses.

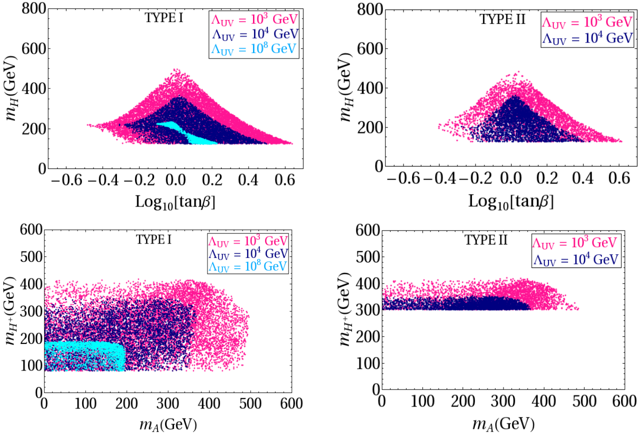
$$\Delta T = \frac{1}{16\pi \sin^2\theta_w M_W^2}\left[F(m_{H^+}^2, m_H^2) + F(m_{H^+}^2, m_A^2) - F(m_H^2, m_A^2)\right] ,$$

$$F(x,y) = \begin{cases} \frac{x+y}{2} - \frac{xy}{x-y}\ln\left(\frac{x}{y}\right) & \text{for } x \neq y , \\ 0 & \text{for } x = y . \end{cases}$$

$$\Delta T = 0.05 \pm 0.12 ,$$

- If, $m_H \approx m_A$ then ΔT severely restricts the splitting between charged and neutral scalar masses.
- For Type II models $m_H^+ > 300$ GeV due to $b \to s\gamma$.
  For Type I models, $m_H^+ > 80$ GeV from direct search limit.
- LHC Higgs data $\Rightarrow$ Alignment limit, $\beta - \alpha = \pi/2$.

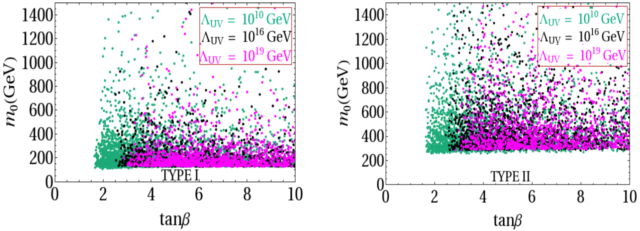
Exact Z_2 symmetric 2HDM



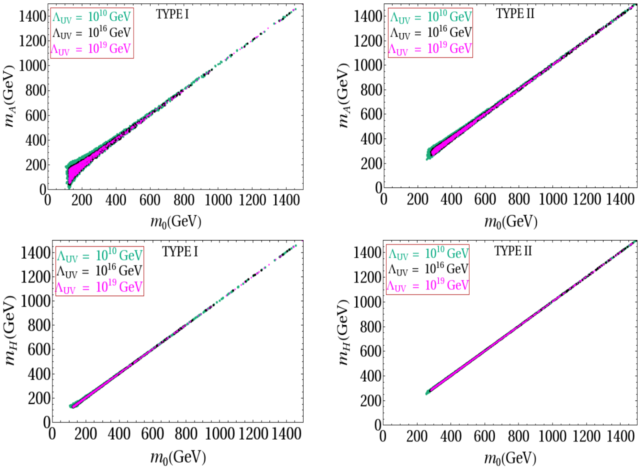
- Potential is not stable until Planck scale.
- Type I models remain stable up to a maximum of 10^8 GeV whereas Type II models can only be stable up to 10^4 GeV due to charged Higgs mass bound.
- $\tan\beta$ is bounded depending upon the energy scale $\Lambda_{\rm UV}$, up to which stability is demanded.

Softly Broken Z_2 symmetric 2HDM

- Absolute stability until the Planck scale can only be achieved for softly broken Z_2 symmetric case but with certain conditions which we named as the *stable alignment limit*.
- Stability of the 2HDM potential up to a cut-off scale $\Lambda_{\rm UV}$ yield a lower bound on $\tan\beta$ and eventually on m_0 (or equivalently on β_5 for $m_0^2 = \frac{1}{2}\beta_5 v^2$).
- $\tan\beta \geq 3$ and $m_0 \geq 120(280)$ GeV for Type I(II).
- The nonstandard scalar masses are all correlated to the soft breaking mass parameter m_0.
- The conclusion do not crucially depend upon top quark pole mass.

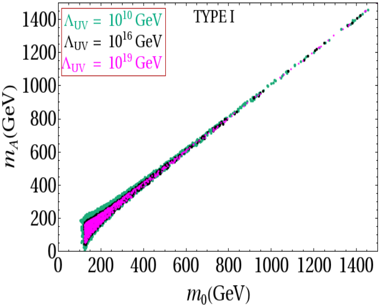


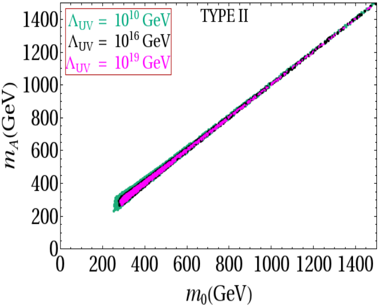
Softly Broken Z_2 symmetric 2HDM



Theoretical Explanation for mass correlation

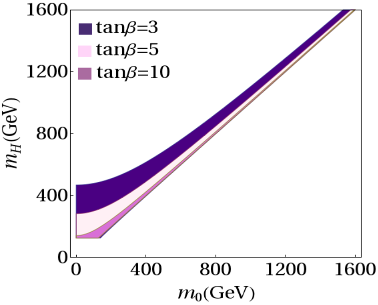
- The evolution of λ_5 is proportional to itself and any initial nonzero value of λ_5 will cause it to grow with energy.
- $\lambda_5 \approx 0$ leads to $U(1)$ limit.
- $\lambda_5 \approx 0$ implies $\beta_5 \approx \beta_6$ and hence, $m_A^2 \approx m_0^2$.

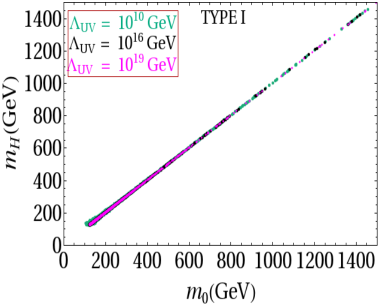




Theoretical Explanation for mass correlation

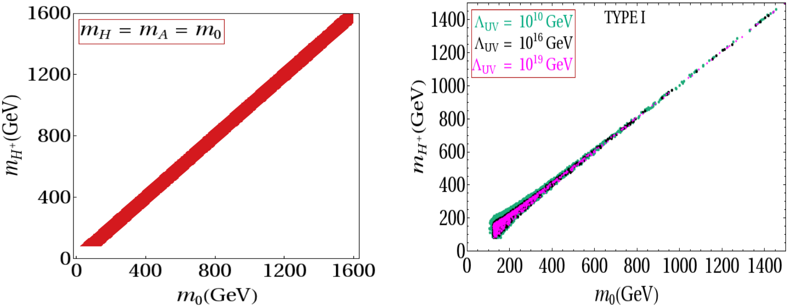
- The unitarity and stability conditions at the electroweak scale imply, [D. Das '15]
  $0 < (m_H^2 - m_0^2)(\tan^2\beta + \cot^2\beta) + 2m_h^2 < \frac{32\pi v^2}{3}$.
- For $\tan\beta$ away from unity, the inequality renders a degeneracy between m_H and m_0.





Theoretical Explanation for mass correlation

- With $m_H^2 \approx m_A^2 \approx m_0^2$, $\Delta T = \frac{1}{8\pi \sin^2\theta_w M_W^2} F\left(m_{H+}^2, m_0^2\right)$.
- $F\left(m_{H+}^2, m_0^2\right)$ restricts the splitting $|m_{H+}^2 - m_0^2|$, the experimental limit on ΔT imparts the degeneracy between m_{H+} and m_0.



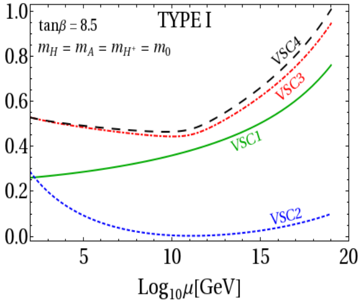
- Thus, $m_0^2 \approx m_A^2 \approx m_H^2 \approx m_{H+}^2$.
- The lower limit of charged Higgs mass for Type II (300 GeV) and limit on CP-even Higgs ($> m_h$) for Type I sets the limit on m_0.

Theoretical Explanation for $\tan\beta$ limit

- ϕ_2 gives masses to up-type quarks $\Rightarrow$ λ_2 will face the negative pull of top Yukawa copling $\to h_t = \frac{\sqrt{2}m_t}{(v\sin\beta)}$.
- In the limit of *exact degeneracy*, RG running of λ_2 is similar to SM Higgs self-coupling λ except some extra scalar contribution.

$$\mathcal{D}\lambda_2 = 16\lambda_2^2 - 3\left(3g^2 + g'^2\right)\lambda_2 + \frac{3}{4}\left(3g^4 + g'^4 + 2g^2g'^2\right) + 12h_t^2\lambda_2 - 12h_t^4 .$$

- Not enough to overcome the negative pull of top Yukawa $\Rightarrow$ lower limit in $\tan\beta$.



Theoretical Explanation for $\tan\beta$ limit

Deviation in $m_H^{\pm}$ from degenerate value relax the $\tan\beta$ limit.

$$m_0 = m_A = m_H\,, \quad \text{and,} \quad m_{H^+}^2 = m_0^2 + \Delta\,.$$

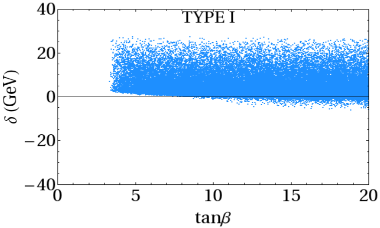
In this limit,

$$\lambda_1 = \lambda_2 = \frac{m_h^2}{v^2}\,, \quad \lambda_3 = \lambda_2 + \frac{2\Delta}{v^2}\,, \quad \lambda_4 = -\frac{2\Delta}{v^2}\,, \quad \lambda_5 = 0\,.$$

Using these, we can rearrange the terms that appear on the RHS in the RG equation for λ_2, to obtain

$$\begin{aligned}\mathcal{D}\lambda_2 \quad &= \quad 14\lambda_2^2 + 2\left(\lambda_2 + \frac{2\Delta}{v^2}\right)^2 - 3\left(3g^2 + {g'}^2\right)\lambda_2 \\ &\quad + \frac{3}{4}\left(3g^4 + {g'}^4 + 2g^2{g'}^2\right) + 12h_t^2\lambda_2 - 12h_t^4\,.\end{aligned}$$

Extra scalar contribution aids to lower value of $\tan\beta$.



Summary

- 2HDM scalar potential with an exact Z_2 symmetry is unable to maintain stability after 10^8 GeV (10^4 GeV) in the Type I (II) case. To ensure stability up to the Planck scale, Z_2 symmetry needs to be broken softly.
- By demanding high scale stability in the presence of a soft breaking, we are led to a situation where the symmetry of the potential is enhanced from softly broken Z_2 to softly broken $U(1)$.
- To have stability up to very high energies ($\gtrsim 10^{10}$ GeV), all the nonstandard masses need to be nearly degenerate: $m_0 \approx m_A \approx m_H \approx m_{H^+}$. Thus, there is only one nonstandard mass parameter that governs the 2HDM in the *stable alignment limit*.
- The value of $\tan\beta$ is bounded from below.

Thank you