Perspectives for Detecting Lepton Flavour Violation in Left-Right Symmetric Models

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Based on arXiv:1611.07025

in collaboration with Cesar Bonilla, Manuel E. Krauss and Werner Porod

ABHM Meeting December 2016



December 16, 2016

WHY LEFT-RIGHT Symmetric Models?

MOTIVATION

Automatic neutrino masses

• Combination both type-I and II seesaw

[R. Mohapatra & G. Senjanovic (1979), ...]



[H. Fritzsch & P. Minkowski (1975), J. Pati & A. Salam (1973), C. Aulakh, et. al. (hep-ph/9712551), ...]

• SO(10)
$$\xrightarrow{M_{\text{GUT}}} (\mathcal{G}_{\text{PS}} \longrightarrow) \mathcal{G}_{\text{LR}} \xrightarrow{v_R} \mathcal{G}_{\text{SM}}$$

$$\mathcal{G}_{PS} = SU(4) \times SU(2)_L \times SU(2)_R$$
$$\mathcal{G}_{LR} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

Strong-CP problem

[A. Maiezza & M. Nemevšek (arXiv:1407.3678), ...]

• Potentially solved by $\mathcal C$ & $\mathcal P$ restoration

SO, WHAT'S THE PLAN?

• Preliminaries

Parametrising the neutrino sector

- Casas-Ibarra parametrisation
- Our parametrisation
- Lepton flavour violation results
 - Current and future bounds
 - (Effect of $\delta_{\rm CP}$)

Preliminaries

Quick-fire recap from last time:

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$$+ \overline{L_{L}} \left(Y_{L_{1}} \Phi + Y_{L_{2}} \tilde{\Phi} \right) L_{R}$$

Field	Gen.	\mathcal{G}_{LR}
Q_L	3	$({f 3},{f 2},{f 1},rac{1}{3})$
Q_R	3	$({f 3},{f 1},{f 2},rac{1}{3})$
L_L	3	$({f 1},{f 2},{f 1},-1)$
L_R	3	(1 , 1 , 2 ,-1)
Φ	1	(1, 2, 2, 0)

$$\begin{split} \tilde{\Phi} &\equiv -\sigma_2 \Phi^* \sigma_2 \\ \overline{\Psi^C} &= \Psi^T C \,, \qquad C = i \gamma_2 \gamma_0 \end{split}$$

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 \hookrightarrow SARAH model file publicly available

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DETERMINING THE NEUTRINO SECTOR

NEUTRINO MASSES

Neutrino mass matrix from \mathscr{L}_Y

$$\mathcal{M}_{\nu} = \begin{pmatrix} M_L^* & M_D \\ M_D^T & M_R \end{pmatrix}$$

$$M_{L/R} = \sqrt{2} Y_{\Delta_{L/R}} v_{L/R}$$
$$M_D = \frac{1}{\sqrt{2}} (Y_{L_1} v_1 + Y_{L_2} v_2)$$

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IN THE SEESAW APPROXIMATION

$$m_{\nu}^{\text{light}} = U_{\text{PMNS}}^{*} \text{diag}(m_{\nu_{1}}, m_{\nu_{2}}, m_{\nu_{3}}) U_{\text{PMNS}}^{\dagger} \stackrel{!}{=} (M_{L}^{*} - M_{D} M_{R}^{-1} M_{D}^{T})$$

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How to determine model parameters?

- Casas-Ibarra parametrisation
- Brute force fits
- Our parametrisation

Simple procedure to find M_D :

$$M_D = f(m_{\nu}^{\text{light}}, M_L, M_R, R)$$

 \hookrightarrow Require an additional matrix R

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Using the above expression we can than determine M_D as required

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IMPLICATIONS:

- R is an unknown matrix $\in O(3; \mathbb{C})$
- Unrelated to model parameters
- Potentially violates $\mathcal C$ & $\mathcal P$
- Results in an infinite number of degenerate solutions

$$\begin{array}{l} \text{Parity } \mathcal{P}: \\ L_L \leftrightarrow L_R \\ \Delta_L \leftrightarrow \Delta_R \\ \Phi \leftrightarrow \Phi^{\dagger} \end{array} \right\} \xrightarrow{\mathscr{L} \text{ inv.}} \begin{cases} Y_{Q_i} = Y_{Q_i}^{\dagger} \\ Y_{L_i} = Y_{L_i}^{\dagger} \\ Y_{\Delta_L} = Y_{\Delta_R} \end{cases}$$

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CONSEQUENCE: $m_{\nu}^{\text{light}} \stackrel{\mathcal{P}}{=} \left(\frac{v_L}{v_R} M_R^* - M_D M_R^{-1} M_D^* \right)$ $m_{\nu}^{\text{light}} \stackrel{\mathcal{C}}{=} \left(\frac{v_L}{v_R} M_R - M_D M_R^{-1} M_D \right)$

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DIRECT ALGEBRAIC SOLUTIONS

$$Y_{\Delta} \equiv Y_{\Delta_L}^{(*)} = Y_{\Delta_R} = f(M_D, v_L, v_R)$$

[E. Akhmedov and M. Frigerio (hep-ph/0509299)]

OUR PARAMETRISATION

 \mathcal{C} -conserving case re-written as:

$$B = \alpha A - A^{-1}$$
$$= V^* \left(\alpha D_A - D_A^{-1} \right) V^{\dagger}$$

$$B \equiv M_D^{-1/2} m_\nu^{\text{light}} M_D^{-1/2} , \quad \alpha \equiv \frac{v_L}{v_R}$$
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Diagonalise and solve resulting diagonal polynomial matrix equations

$$D_A^{(i,i)} = \frac{D_B^{(i,i)} \pm \sqrt{\left(D_B^{(i,i)}\right)^2 + 4\alpha}}{2\alpha}$$

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Result

$$Y_{\Delta}^{(\pm\pm\pm)} = \frac{1}{2\sqrt{2}v_L} M_D^{1/2} V^* \text{diag}\left(D_B^{(i,i)} \pm \sqrt{\left(D_B^{(i,i)}\right)^2 + 4\alpha}\right) V^{\dagger} M_D^{1/2}$$

Advantages:

- Eightfold degenerate solutions
- **2** Function only of model parameters

LEPTON FLAVOUR VIOLATION

LEPTON FLAVOUR OBSERVABLES

CALCULATED OBSERVABLES:

- $\ell_{\alpha} \rightarrow \ell_{\beta} \gamma$
- $\ell_{\alpha} \rightarrow 3\ell_{\beta}$
- $\ell_{\alpha} \rightarrow \ell_{\beta} \ell_{\gamma} \ell_{\delta}$
- μe conversion

Including all tree and one-loop contributions

LEPTON FLAVOUR OBSERVABLES



LEPTON FLAVOUR OBSERVABLES



TRIPLET-YUKAWA SIGN-CHOICE



• Sufficient to consider **only** two choices

CURRENT AND FUTURE BOUNDS

DIAGONAL M_D



- Scalar tree-level contributions dominate
- Off-diagonal Y_{Δ} vanish at LO

=

$$D_B = \frac{m_{\nu}^{\text{light}}}{M_D} \ll \mathcal{O}\left(\sqrt{\alpha}\right)$$
$$\Rightarrow D_A \simeq \pm \alpha^{-1/2}$$
$$\hookrightarrow A^{(+++)} = V D_A V^T = D_A$$

Diagonal M_D



• Scalar tree-level contributions again dominate

• Off-diagonal Y_{Δ} <u>do not</u> vanish at LO

$$A^{(+-)} = V \begin{pmatrix} \alpha^{-1/2} & 0\\ 0 & -\alpha^{-1/2} \end{pmatrix} V^T = \alpha^{-1/2} \begin{pmatrix} \cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \neq D_A$$

$$M_D = V_{\rm CKM}^{\dagger} M_{\rm up-type} V_{\rm CKM}^*$$



- Additonal flavour violation from CKM elements
- Large hierarchy in Y_{Δ}
- Interplay between γ -penguins and tree-level scalar contributions Example $\mu \to 3e$:

$$\mathcal{M}_{\text{tree-level}} \propto Y_{\Delta}^{(2,1)} Y_{\Delta}^{(1,1)} \qquad \mathcal{M}_{\gamma-\text{peng}} \propto \frac{1}{16\pi^2} \sum_{i} Y_{\Delta}^{(2,i)} Y_{\Delta}^{(i,1)}$$

CONCLUSIONS

- Exploited discrete left-right symmetries to simplify neutrino sector
- Examined current constraints and prospects of probing low-scale left-right symmetries using LFV observables

Also investigated:

- Varying scalar masses
- Neutrino hierarchy & mass scale
- Varying $\delta_{\rm CP}$

BACKUP SLIDES

Effect of δ_{CP}



Effect of δ_{CP}



• For both sign choices off-diagonal Y_{Δ} <u>do not</u> vanish at LO

$$A^{(+-)} = V^* D_A V^{\dagger} \neq D_A$$

Scalar Sector

Field content

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \qquad \Delta_L = \begin{pmatrix} \frac{\delta_L^+}{\sqrt{2}} & \delta_L^{++} \\ \delta_L^0 & -\frac{\delta_L^+}{\sqrt{2}} \end{pmatrix} \qquad \Delta_R = \begin{pmatrix} \frac{\delta_R^+}{\sqrt{2}} & \delta_R^{++} \\ \delta_R^0 & -\frac{\delta_R^+}{\sqrt{2}} \end{pmatrix}$$

with VEVs

$$\begin{split} \phi_1^0 &= \frac{1}{\sqrt{2}} \left(v_1 + \sigma_1 + i\varphi_1 \right) & \delta_L^0 &= \frac{1}{\sqrt{2}} \left(v_L + \sigma_L + i\varphi_L \right) \\ \phi_2^0 &= \frac{1}{\sqrt{2}} \left(v_2 + \sigma_2 + i\varphi_2 \right) & \delta_R^0 &= \frac{1}{\sqrt{2}} \left(v_R + \sigma_R + i\varphi_R \right) \end{split}$$

Scalar Masses

Bi-doublet/SM-like masses

$$m_h^2 \simeq 2\lambda_1 v^2 - \frac{8\lambda_4^2 v^4}{\alpha_3 v_R^2}$$
$$m_A^2 \simeq 2\alpha_3 v_R^2 + 2(\lambda_3 - 2\lambda_2) v^2$$

$$m_H^2 \simeq 2(2\lambda_2 + \lambda_3)v^2 + \frac{\alpha_3}{2}v_R^2$$
$$m_{H^{\pm}}^2 \simeq \frac{1}{4}\alpha_3(v^2 + 2v_R^2)$$

Left/right triplet masses

...

$$\begin{split} m_{H_L}^2 &\simeq \frac{1}{2} \left(\rho_3 - 2\rho_1 \right) v_R^2 & m_{H_R}^2 \simeq 2\rho_1 v_R^2 \\ m_{A_L}^2 &\simeq \frac{1}{2} (\rho_3 - 2\rho_1) v_R^2 & m_{H_L^\pm}^2 \simeq \frac{1}{2} v_R^2 (\rho_3 - 2\rho_1) \,, \\ m_{H_1^{\pm\pm}}^2 &\simeq 2\rho_2 v_R^2 + \frac{1}{2} \alpha_3 v^2 & m_{H_2^{\pm\pm}}^2 \simeq \frac{1}{2} \left((\rho_3 - 2\rho_1) v_R^2 + \alpha_3 v^2 \right) \end{split}$$

LFV BOUNDS & FUTURE SENSITIVITY

LFV Process	Present Bound	Future Sensitivity
$\mu \to e\gamma$	4.2×10^{-13}	6×10^{-14}
$\tau ightarrow e \gamma$	$3.3 imes 10^{-8}$	$\sim 3 \times 10^{-9}$
$ au o \mu \gamma$	4.4×10^{-8}	$\sim 10^{-9}$
$\mu \rightarrow eee$	1.0×10^{-12}	$\sim 3 \times 10^{-16}$
$\tau \to eee$	2.7×10^{-8}	$\sim 5 \times 10^{-10}$
$ au ightarrow \mu \mu \mu$	2.1×10^{-8}	$\sim 4 \times 10^{-10}$
$\tau^- ightarrow e^- \mu^+ \mu^-$	$2.7 imes 10^{-8}$	$\sim 5 imes 10^{-10}$
$\tau^- \to \mu^- e^+ e^-$	$1.8 imes 10^{-8}$	$\sim 3 \times 10^{-10}$
$\tau^- ightarrow \mu^+ e^- e^-$	$1.5 imes 10^{-8}$	$\sim 3 \times 10^{-10}$
$\tau^- \to e^+ \mu^- \mu^-$	1.7×10^{-8}	$\sim 3 \times 10^{-10}$
$\mu^- \rightarrow e^-, \mathrm{Ti}$	4.3×10^{-12}	$\sim 10^{-18}$
$\mu^- \rightarrow e^-, \mathrm{Au}$	7×10^{-13}	-
$\mu^- \rightarrow e^-, \mathrm{Al}$	-	$10^{-16} - 3 \times 10^{-17}$
$\mu^- \to e^-, \mathrm{SiC}$	-	10^{-14}

Model Parameters and Mass Spectrum

Model Parameters					
λ_1	0.13	v_L	$10^{-10}1{\rm GeV}$		
λ_2	1.0	v_R	$20\mathrm{TeV}$		
λ_3	1.0	an eta	10^{-4}		
λ_4	0	α_1	0		
ρ_1	3.2×10^{-4}	α_2	0		
ρ_2	$2.5 imes 10^{-4}$	α_3	2.0		
ρ_3	$1.8 imes 10^{-3}$	β_1	0		
$ ho_4$	0	β_2	$3.83 imes 10^{-4}$		
μ_1^2	$7.87 imes 10^3 { m GeV^2}$	β_3	0		
μ_2^2	$-2.00\times 10^4{\rm GeV^2}$	μ_3^2	$1.28\times 10^5{\rm GeV^2}$		
Resulting Mass Spectrum					
m_h	$125.5{ m GeV}$	m_H	$20{\rm TeV}$		
m_A	$20\mathrm{TeV}$	$m_{H^{\pm}}$	$20\mathrm{TeV}$		
m_{H_L}	$482{ m GeV}$	m_{H_R}	$506{ m GeV}$		
m_{A_L}	$482{ m GeV}$	$m_{H_I^{\pm}}$	$512{ m GeV}$		
$m_{H_1^{\pm\pm}}$	$511{ m GeV}$	$m_{H_2^{\pm\pm}}^L$	$541{ m GeV}$		
M_{W_R}	$9.37{ m TeV}$	M_{Z_R}	$15.7\mathrm{TeV}$		